# Interconnection and Damping Assignment Passivity-Based Control for Port-Hamiltonian mechanical systems with only position measurements

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*Abstract*— **A dynamic extension for position feedback of port-Hamiltonian mechanical systems is studied. First we look at the consequences for the matching equations when applying Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC). Then we look at the possibilities of asymptotically stabilizing a class of port-Hamiltonian mechanical systems without having to know the velocities, as once presented for Euler-Lagrange (EL) systems. Here it is shown how the idea of damping injection by dynamic extension works when shaping the total energy in the port-Hamiltonian framework.**

## I. INTRODUCTION

The successful application of IDA-PBC for mechanical systems has been shown in recent work [1], [2] and [3], for systems where physical damping (e.g. friction) is neglected. The advantage of IDA-PBC is the possibility of shaping the total energy of a system, which is especially useful for underactuated system. These systems usually require kinetic and potential energy shaping in order to achieve the desired stable equilibrium points. Total energy shaping has also been shown in [4] for a class of mechanical EL systems. In this paper we restrict ourselves to total energy shaping for port-Hamiltonian (mechanical) systems.

In this paper we want to study the idea of a dynamic extension for port-Hamiltonian systems as presented in [5] for EL systems. The application of a dynamic extension for mechanical EL systems allows to inject damping to the system, making it unnecessary to know the velocities for damping assignment. We want to combine this idea with total energy shaping, realized by applying IDA-PBC. In [6] a dynamic extension for port-Hamiltonian systems was already presented. They also showed that velocity measurements can be omitted, but they do that only for potential energy shaping. They also interconnect the system with the controller through the ports. The idea of a dynamic extension for port-Hamiltonian systems for total energy shaping has been presented in [7] and in [8], where a port-Hamiltonian plant was interconnected to a port-Hamiltonian controller. In contrast to what is done in [5] these controllers, or dynamic extensions, depend only on the controller coordinates $q_c$. The result is a closed-loop system where the interconnection is realized through the ports. A dynamic extension for output stabilization was presented in [9] for a class of nonholonomic Hamiltonian systems. Here the authors realized a dynamic extension by adding an integrator to the system via a generalized canonical transformation. After this they derived an output feedback stabilization method. In the following we want to explore the idea of controllers with potential energy depending on both system coordinates $q$ and $q_c$. This is done for a class of systems where velocity measurements are not necessary for stabilization. Section II shortly recaps PBC for EL systems and the application of a dynamic extension in this case. A short summary is also given of how IDA-PBC works. Section III shows how a dynamic extension, as described in [5], is realized for port-Hamiltonian systems and what the consequences are when this type of dynamic extension is used. This section first looks at the matching conditions [10] when applying IDA-PBC and the effect that the dynamic extension has on these conditions. Then we explore the possibilities of asymptotically stabilizing a system without having to know the velocity $\dot{q}$ as presented in [5] for EL mechanical systems. The application of IDA-PBC on port-Hamiltonian systems with dynamic extension is shown for two examples in section IV. In the final section concluding remarks are given.

## II. PASSIVITY-BASED CONTROL

### *Euler-Lagrange systems*

In [5] it is shown how for EL mechanical systems the potential energy is shaped to achieve the desired equilibrium points. It is also shown how with a dynamic extension the system can be asymptotically stabilized when velocities are not measured. A dynamical system with generalized coordinates $q = (q_1, ..., q_n)^\top$ and external forces $Q$ can be described by the EL equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}(q,\dot{q})\right) - \frac{\partial L}{\partial q}(q,\dot{q}) = Q \tag{1}$$

where $L(q, \dot{q})$ is the Lagrangian function defined to be the difference between the system kinetic energy, $T(q, \dot{q})$, and

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the potential energy, $V(q)$. With this equation we define a plant system

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}(q,\dot{q})\right)-\frac{\partial L}{\partial q}(q,\dot{q})=M_p u \quad (2)$$

with input matrix $M_p$ and input $u$. An EL controller can be defined in the same way with the only difference that the potential energy of the controller depends on both plant coordinates $q$ and controller coordinates $q_c$,

$$\frac{d}{dt}\left(\frac{\partial L_c}{\partial \dot{q}_c}(q,q_c,\dot{q}_c)\right)-\frac{\partial L_c}{\partial q_c}(q,q_c,\dot{q}_c)+\frac{\partial \mathcal{F}_c}{\partial \dot{q}_c}(\dot{q}_c)=M_c u_c \quad (3)$$

Here $L_c(q,q_c,\dot{q}_c)$ is the controller Lagrangian, $M_c$ the controller input matrix and $u_c$ the controller input. The controller also has dissipation energy $\mathcal{F}_c(\dot{q}_c)$. In [5] the feedback interconnection between plant and controller is established by

$$M_p u=-\frac{\partial V_c}{\partial q_m}(q,q_c), \quad (4)$$

$q_m$ being the measurable coordinates of $q$. The measurable output $q_m$ enters into the dynamic extension via $\frac{\partial V_c}{\partial q_c}(q_m,q_c)$. By an appropriate choice of the controller energy $V_c$ the potential energy of the plant is shaped such that the desired equilibrium point $q^*$ is realized. In [5] it is also shown under which conditions the plant can be asymptotically stabilized by the dynamic extension. Velocity measurement is not necessary since damping is injected through the controller.

*Port-Hamiltonian systems*

One advantage of IDA-PBC is the possibility of shaping the total energy [1] of underactuated systems. If a conservative port-Hamiltonian mechanical system is described by

$$\dot{x} = J(x)\frac{\partial H}{\partial x}(x)+g(x)u \quad (5)$$

$$y = g(x)^\top\frac{\partial H}{\partial x}(x) \quad (6)$$

where $x$ are the states of the system: $x=(q,p)^\top$ the vector of generalized configuration coordinates $q=(q_1,...,q_n)$ and generalized momenta $p=(p_1,...p_n)^\top$, interconnection matrix $J(x)$ and input matrix $g(x)$. The Hamiltonian $H(x)$ is defined as the kinetic plus potential energy of the system

$$H(q,p)=\frac{1}{2}p^\top M(q)^{-1}p+V(q) \quad (7)$$

with $M$ being the plant mass matrix. By applying IDA-PBC we want to achieve a port-Hamiltonian system with a new interconnection matrix $J_d(x)$ and desired Hamiltonian $H_d$,

$$\dot{x}=(J_d(x)-R_d(x))\frac{\partial H_d}{\partial x}(x) \quad (8)$$

$J_d(x)$ usually (for mechanical systems) takes the form

$$J_d=\begin{bmatrix} 0 & M(q)^{-1}M_d(q)\\ -M_d(q)M(q)^{-1} & J_2(x)\end{bmatrix} \quad (9)$$

with $J_2(x)$ a free to choose skew symmetric matrix. Damping is assigned through the damping matrix $R_d\geq 0$. The new Hamiltonian $H_d(x)$ has the desired equilibrium points $q^*$,

$$H_d(q,p)=\frac{1}{2}p^\top M_d(q)^{-1}p+V_d(q) \quad (10)$$

$M_d$ being the new mass matrix. This results in a partial differential equation (PDE) to be solved

$$g(x)^\perp[(J_d(x)-R_d(x))\frac{\partial H_d}{\partial x}(x)-J(x)\frac{\partial H}{\partial x}(x)]=0 \quad (11)$$

with $g^\perp g=0$, which can be divided into a kinetic energy PDE and a potential energy PDE. These PDEs are also called the matching equations (or matching conditions) [10]. The input signal is naturally decomposed in two terms [1]

$$u=u_{es}(q,p)+u_{di}(q,p) \quad (12)$$

where the first term shapes the energy and the second term injects damping. To asymptotically stabilize the system damping is injected through the damping matrix $R_d$. The energy shaping input signal becomes

$$u_{es}=(g(x)^\top g(x))^{-1}g(x)^\top[J_d(x)\frac{\partial H_d}{\partial x}(x)-J(x)\frac{\partial H}{\partial x}(x)] \quad (13)$$

and the second term

$$u_{di}=-R_d(x)g(x)^\top\frac{\partial H_d}{\partial x}(x) \quad (14)$$

The system described by (8) does not describe physical damping present in the system. Taking the physical damping into consideration results in an additional condition to be satisfied, the *dissipation condition* [11]. In this paper we only look at systems where physical damping is neglected.

## III. Dynamic extension for port-Hamiltonian systems

*Realization*

As mentioned in the introduction a dynamic extension for port-Hamiltonian systems has already been presented in [7], [6] and [8]. However, the interconnection between plant and controller was made through the ports. In this paper we want to interconnect the systems through an appropriate new, desired, Hamiltonian $\tilde{H}_d(q,p,q_c,p_c)$. To be more precise, it is in the new potential energy $\tilde{V}_d(q,q_c)$ where this interconnection is described. In the original setup applying IDA-PBC results in a solution $H_d(q,p)$ which has the desired equilibrium points. In the setup proposed in this paper this solution is still present, but the interconnection between plant and controller is also described in the new Hamiltonian. This results in the extended closed-loop system

$$\begin{bmatrix}\dot{x}\\ \dot{x}_c\end{bmatrix}=\begin{bmatrix} J_d(x) & 0\\ 0 & J_c(x_c)-R_c\end{bmatrix}\begin{bmatrix}\frac{\partial \tilde{H}_d}{\partial x}\\ \frac{\partial \tilde{H}_d}{\partial x_c}\end{bmatrix} \quad (15)$$

with

$$\tilde{H}_d(q,p,q_c,p_c) = \frac{1}{2}p^\top M_d(q)^{-1}p+\frac{1}{2}p_c^\top M_c^{-1}(q_c)p_c+ +\tilde{V}_d(q,q_c) \quad (16)$$

in which $x_c = [q_c, p_c]^\top$, $M_c$ is the controller mass matrix, $J_c = -J_c^\top$ is the controller interconnection matrix and $R_c = R_c^\top \geq 0$ is the controller damping matrix. The potential energy $\tilde{V}_d$ is not entirely free to choose since we now have the matching condition

$$g(x)^\perp [J_d(x) \frac{\partial \tilde{H}_d}{\partial x}(x, x_c) - J(x) \frac{\partial H}{\partial x}(x)] = 0 \qquad (17)$$

ensuring that the closed-loop equations describing $\dot{x}$ and (8) match.

*Influence on matching conditions*

At first sight it may seem that the matching conditions (11) and (17) are almost similar, if $R_d$ in (11) is neglected. However, in the conditions of (17) extra terms are present caused by the states interconnecting plant and controller. Since now we have additional terms, is it possible that the extension is helpful when solving the resulting PDEs? First we assume the system (8) to be of the form

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u \qquad (18)$$

The matching condition can be divided into a kinetic energy PDE and a potential energy PDE. For the dynamic extension only the potential energy of the controller depends on plant states, so we only look at the potential energy PDE. For the system (18), resulting in a closed-loop system of the form (15), this potential energy PDE becomes

$$G^\perp [\frac{\partial V}{\partial q}(q) - M_d(q) M(q)^{-1} \frac{\partial \tilde{V}_d}{\partial q}(q, q_c)] = 0 \qquad (19)$$

If we want to solve this PDE we have to solve $\frac{\partial \tilde{V}_d}{\partial q}(q, q_c)$ for

$$G^\perp M_d(q) M(q)^{-1} \frac{\partial \tilde{V}_d}{\partial q}(q, q_c) = G^\perp \frac{\partial V}{\partial q}(q) \qquad (20)$$

In the original setup (no dynamic extension) we would have to solve

$$G^\perp M_d(q) M(q)^{-1} \frac{\partial V_d}{\partial q}(q) = G^\perp \frac{\partial V}{\partial q}(q) \qquad (21)$$

Notice that it does not matter whether we have $V_d(q)$ or $\tilde{V}_d(q, q_c)$ since the solution of both is fixed by $V(q)$. Both (20) and (21) have the same right hand term, forcing the same solution in both situations. We can also define for simplicity $G$ as in [11] to have the form

$$G = \begin{bmatrix} 0_{(n-m)\times m} \\ I_m \end{bmatrix} \qquad (22)$$

for a system with actuated and unactuated coordinates $q = (q_u, q_a)$. For underactuated systems $V_c(q, q_c)$ cannot have influence on the unactuated coordinates $q_u$. An extra term could influence the efforts on the actuated coordinates $q_a$, but this freedom was already present because of actuation. It becomes clear that a controller with potential energy depending on both plant and controller coordinates does not influence the solvability of the matching equations. This was shown in [13] for the general case.

*Asymptotic stabilization*

One of the nice properties of dynamic extension applied to EL systems is the ability to inject damping without having to know the velocity $\dot{q}$. The necessary damping to asymptotically stabilize the system was provided by the damping of the controller. The conditions to asymptotically stabilize a system by dynamic extension presented in [5] are somewhat different in the port-Hamiltonian case since now we also have to satisfy the matching condition (17). The following proposition is limited to two kind of systems:

- Systems that need only potential energy shaping (e.g. fully actuated systems), or
- Systems with constant mass matrix $M$.

For the first type of systems the kinetic energy does not have to be shaped (can stay the same) and $M_d(q)$ can be chosen equal to $M(q)$. Because only the potential energy is shaped velocity measurements are not necessary for stabilization. The same idea applies for the second type of systems. Since $M$ is constant, $M_d$ can be chosen constant too and the kinetic energy PDE disappears. In both cases the free matrix $J_2$ can be chosen equal to zero making $u_{es}$, see (12), depend only on $q$ measurements.

***Proposition** 1:* A dynamic extension for port-Hamiltonian systems resulting in the closed-loop system (15) asymptotically stabilizes the plant (8) belonging to the class described above if

1) The Hamiltonian $\tilde{H}_d$ has its minimum $\frac{\partial \tilde{H}_d}{\partial q}(q, q_c) = 0$ in $q = q^*, q_c = q_c^*$.
2) The matching condition (17) is satisfied.
3) For $\frac{\partial \tilde{V}_d}{\partial q_c}(q, q_c^*) = 0$, we have that $q$ is constant.

***Proof.*** The desired equilibrium point is realized if the new Hamiltonian (the new energy function) $\tilde{H}_d$ has its minima at the equilibrium point $q = q^*$. The second condition is necessary for the closed-loop system equations describing $\dot{q}$ and $\dot{p}$ and the (uncontrolled plant) to match, [10], [12]. The last condition comes from the in [5] presented *dissipation propagation* condition. Asymptotic stability is proved invoking LaSalle's invariance principle for the closed-loop system (15) where

$$\frac{d}{dt}\tilde{H}_d = -\dot{q}_c^\top R_c \dot{q}_c \qquad (23)$$

From LaSalle's principle we know that for asymptotic stability we need $\frac{d}{dt}\tilde{H}_d \leq 0$, being equal to zero only for the equilibrium points. The function (23), which is negative semidefinite, is equal to zero only when $\dot{q}_c = 0$, meaning that $q_c$ must be a constant. The equilibrium point $q_c^*$ of the controller is found by

$$\frac{\partial \tilde{V}_d}{\partial q_c}(q, q_c) = 0 \qquad (24)$$

For a constant $q_c$ $q$ should also be constant to satisfy (24). The coordinates $(q, q_c)$ are constants only if they are also the equilibrium points. □

In [5] it is mentioned that the kinetic energy plays no role in stabilizing the system but may however affect transient response. For this reason they are able to define controllers (with some abuse of terminology also called EL controllers) that depend only on potential and dissipative energy. For port-Hamiltonian systems such a controller could be a special case of the dynamic extension because the dynamics do not depend on $p_c$. The controller dynamics is described by interconnection and damping matrices:

$$J_c = 0, \qquad R_c = \begin{bmatrix} \tilde{R}_c^{-1} & 0 \\ 0 & 0 \end{bmatrix} \tag{25}$$

resulting in the (special case) port-Hamiltonian system

$$\dot{q}_c = -\tilde{R}_c^{-1} \frac{\partial \tilde{V}_d}{\partial q_c}(q, q_c) \tag{26}$$

with the interconnection between plant and controller being described in the new potential energy $\tilde{V}_d(q, q_c)$. The result is also a port-Hamiltonian closed-loop system since we have

$$\begin{bmatrix} \dot{x} \\ \dot{q}_c \end{bmatrix} = \begin{bmatrix} J_d & 0 \\ 0 & -\tilde{R}_c^{-1} \end{bmatrix} \begin{bmatrix} \frac{\partial \tilde{H}_d}{\partial x} \\ \frac{\partial \tilde{H}_d}{\partial q_c} \end{bmatrix} \tag{27}$$

In all cases the interconnection between plant and controller is established by the new potential energy function $\tilde{V}_d(q, q_c)$.

From an applications point of view this can be interesting since the dynamic extension eliminates the need to measure the velocities to achieve damping injection. Actually we are omitting the term $u_{di}$ of (12). This is especially attractive when stabilization and costs are important, since now less sensors are necessary. However, a tradeoff with performance is inevitable, as will be shown in the examples in the next section.

## IV. EXAMPLES

In this section two examples are studied to show how the dynamic extension works for port-Hamiltonian systems. The first system is the TORA[1], used in [5]. In this example only the potential energy needs to be shaped. The second example is an inertia wheel pendulum where the *total* energy needs to be shaped. The application of IDA-PBC on this system was presented in [1], [3]. In the following examples $k_p$, $k_c$ and $k_d$ are control constants. The systems are modeled as presented in (18) with control input

$$u = (G^\top G)^{-1} G^\top [\frac{\partial H}{\partial q} - M_d M^{-1} \frac{\partial \tilde{H}_d}{\partial q}] \tag{28}$$

Some simulation results are also presented to show the time response of the systems.

[1]A translational oscillator with an attached eccentric rotational proof mass actuator.

*TORA*

The TORA system is described by

$$\begin{aligned} M(q) &= \begin{bmatrix} M_{cart} + m & -ml\cos q_2 \\ -ml\cos q_2 & I + ml^2 \end{bmatrix} \\ V(q) &= \frac{1}{2} k q_1^2 \\ G &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

with $M_{cart}$ being the cart mass, a proof mass actuator with mass $m$ and inertia $I$ at a distance $l$ from its rotational axis. The system is shown in figure 1 , gravitational forces being neglected because motion takes place in an horizontal plane. This example only requires potential energy shaping

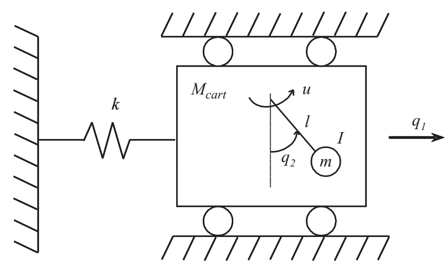


Fig. 1. Rotational/translational proof mass actuator.

and $M_d(q)$ can be chosen equal to $M(q)$. In this case only the potential energy PDE (19) is important which becomes

$$G^\perp [\frac{\partial V}{\partial q}(q) - \frac{\partial \tilde{V}_d}{\partial q}(q, q_c)] = 0$$

with $G^\perp = [1 \; 0]$ and is satisfied by

$$\begin{aligned} \tilde{V}_d(q, q_c) &= \frac{1}{2} k q_1^2 + \frac{1}{2} k_p q_2^2 + \frac{1}{2} k_c (q_2 - q_c)^2 \\ \mathcal{F}_c(\dot{q}_c) &= \frac{1}{2} k_d \dot{q}_c^2 \end{aligned}$$

The control signal (28) realizes the closed-loop system of the form (27). Although the controller energy part is somewhat different than the one used in [5], it still is possible to achieve similar results as in the EL case. The difference is now that we are working in the port-Hamiltonian framework and we have a different extended closed-loop interconnection matrix than the one presented in [7], [8]. The results for the TORA are shown in figure 2. These results are similar to the ones obtained in [5], where the time response is simulated for saturated EL controllers. There the response converged faster, but with higher inputs. Notice in the figure that smaller deviations for $q_2$ are accomplished compared to the situation where velocity measurements are used.

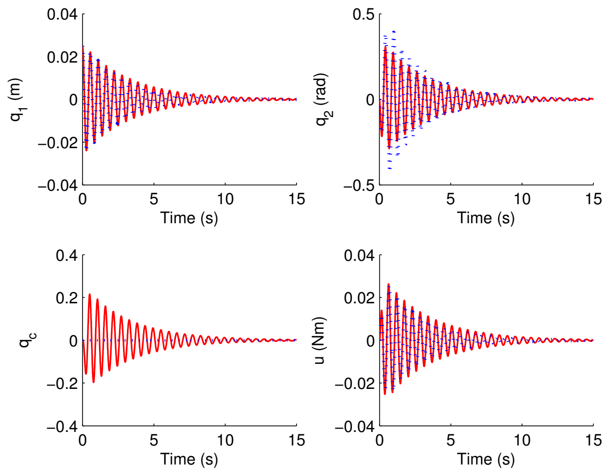


Fig. 2. Trajectories for the TORA system. Initial conditions: $[q(0)\ p(0)] = [0.025\ 0\ 0\ 0]$. The dotted lines represent the results when there is damping input $u_{di}$ (no extension)

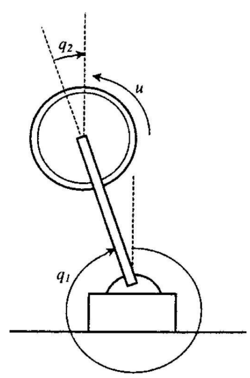


Fig. 3. Inertia wheel pendulum.

*Inertia wheel pendulum*

The inertia wheel pendulum, figure 3, is described in the form of (18) by

$$
\begin{aligned}
M &= \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix} \\
V(q) &= mgl(\cos q_1 - 1), \\
G &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\end{aligned}
$$

where $I_1$ is the moment of inertia of the pendulum, $I_2$ the moment of inertia of the wheel, $m$ is the pendulum mass, $g$ the gravity constant and $l$ the length of the pendulum. In [3] IDA-PBC was applied on this example, with input signal (12). We now want to use the energy shaping input $u_{es}$ presented there and omit the damping injection signal $u_{di}$, which depends on velocity measurements. From [3] we have

$$
\begin{aligned}
M_d &= \begin{bmatrix} a_1 & a_2 \\ a_2 & a_3 \end{bmatrix} \\
V_d(q) &= \frac{mglI_1}{a_1 - a_2} \cos q_1 + \\
&\quad + \frac{P}{2}[q_2 - q_2^* + \gamma_1(q_1 - q_1^*)]^2
\end{aligned}
$$

Here $q^*$ are the desired equilibrium points, $P$ and $\gamma_1$ are constants. In the new setup the potential energy has to be changed such that we have the plant interconnected with the controller and also satisfying the matching condition (17) which now can be written as

$$
G^\perp[\frac{\partial V}{\partial q} - M_d M^{-1}\frac{\partial \tilde{V}_d}{\partial q}] = 0 \tag{29}
$$

In order to achieve this it is proposed to have the new potential energy

$$
\begin{aligned}
\tilde{V}_d(q, q_c) = &\frac{mglI_1}{a_1 - a_2}\cos q_1 + \\
&+ \frac{P}{2}[q_2 - q_2^* + \gamma_1\gamma_2(q_1 - q_1^*) + \frac{1}{2}k_c(q_2 - q_c)]^2
\end{aligned}
$$

and dissipation energy

$$
\mathcal{F}_c(\dot{q}_c) = \frac{1}{2}k_d\dot{q}_c^2
$$

with $\gamma_2 = 1 + k_c$, a constant necessary to satisfy the matching condition. The control signal (28) results in a closed-loop system (27). In [3] two equilibrium points were studied for $q_1$, the hanging position $q^* = (\pi, 0)^\top$ and the upright position $q^* = (0, 0)^\top$. For the hanging position $M_d$ is chosen equal to $M$ (actually resulting in only potential energy shaping) and for the upright position we have $(a_1, a_2, a_3) = (1, 2, 5)$, as in [3]. The results for both desired equilibrium points are shown in figures 4 and 5.

***Remark***. For both examples simulation results are shown for only the closed-loop system of the form described by (27). If in addition to the potential energy also kinetic energy is assigned to the controller, then the performance (time response) either deteriorates (larger deviations, larger input signals) or stays the same, provided that $M_c$ is small enough.

## V. Concluding Remarks

One of the important advantages of a dynamic extension is the possibility of injecting damping without having to know the velocities of the system in order to asymptotically stabilize it. Damping was injected through the damping of an appropriate (virtual) controller. This paper showed how this could be accomplished for port-Hamiltonian systems, with an interconnection not made through the ports, as is usually done. The interconnection is established in the new desired energy function. In short, it is possible to shape the total energy of a mechanical system and asymptotically stabilize

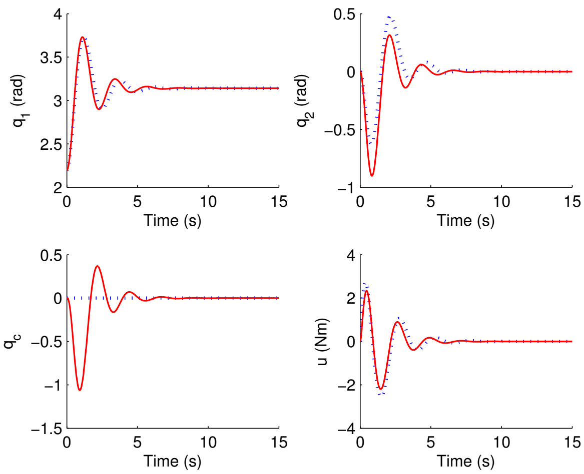


Fig. 4. Trajectories for the inertia wheel pendulum, stabilization of hanging position. Initial conditions: $[q(0)\ p(0)] = [0.7\pi\ 0\ 0\ 0]$. The dotted lines represent the results when there is damping input $u_{di}$ (no extension).

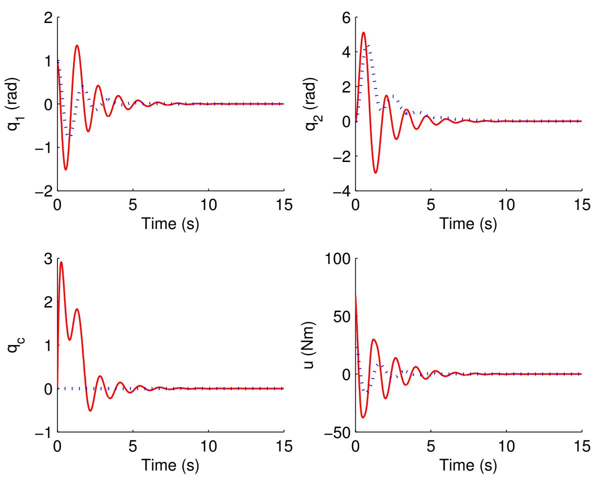


Fig. 5. Trajectories for the inertia wheel pendulum, stabilization of upright position. Initial conditions: $[q(0)\ p(0)] = [0.3\pi\ 0\ 0\ 0]$. The dotted lines represent the results when there is damping input $u_{di}$ (no extension).

it without having to measure the velocities. For applications this could be interesting since it means that velocity sensors are not necessary.

Two examples, for which the dynamic extension makes velocity measurements unnecessary, were shown. The dissipation energy of the controller asymptotically stabilizes the systems if this dissipation was propagated to the other coordinates. We finalize by giving a remark about the TORA system. Although the results show convergence to the desired points, further improvement of the performance can possibly be achieved by another choice of the function $\tilde{H}_d$.

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